

| Configuration | Main Cavity Only | Main Cavity + Harmonic Cavity for Bunch Lengthening | Main Cavity + Harmonic Cavity for Bunch Shortening |
|---|---|---|--|
| Constraints | $k = \phi_N = 0$, $\cos\phi_s > 0$, & $U_0 = eV_0 \sin\phi_s$ | 1.) Remove the ϕ^2 term in the potential, 2.) Prevent minibuckets; remove ϕ^3 term: $k = \frac{1}{N} \sqrt{\cos^2 \phi_s + \frac{1}{N^2} \sin^2 \phi_s}$ $\phi_N = \frac{1}{N} \tan^{-1} \left(\frac{1}{N} \tan\phi_s \right)$ | 1.) Maximize the voltage gradient: Set $N\phi_N = 0$, 2.) k is a free parameter, but minibuckets will form for large k depending on the value of $\sin\phi_s$. |
| Effective Potential $U(\phi)$ | $-\cos(\phi + \phi_s) + \cos\phi_s$ $-\phi \sin\phi_s$ | $(1 - \frac{1}{N^2}) [\cos\phi_s - \phi \sin\phi_s] - \cos\phi_s [\cos\phi - \frac{\cos N\phi}{N^2}] + \sin\phi_s \left[\sin\phi - \frac{\sin N\phi}{N^3} \right]$ | $-\cos(\phi + \phi_s) + \cos\phi_s$ $-\frac{k}{N} [\cos N\phi - 1] - \phi \sin\phi_s$ |
| $U(\phi)$ for $\phi \approx 0$ | $\frac{\cos\phi_s}{2} \phi^2$ | $[N^2 - 1] \frac{\cos\phi_s}{24} \phi^4$ | $\frac{\cos\phi_s}{2} \phi^2 \left(1 + \frac{Nk}{\cos\phi_s} \right)$ |
| Synchrotron Tune | $v_{s0} = \left[\frac{\alpha h \cos\phi_s}{2\pi} \frac{eV_0}{E} \right]^{1/2}$ | $v_{s0} \frac{2\pi^{3/2}}{\Gamma^2(1/4)} \left[\frac{N^2 - 1}{6} \right]^{1/2} \phi_{\max}$ | $v_{s0} \left(1 + \frac{Nk}{\cos\phi_s} \right)^{1/2}$ |
| Bunch Length | $\sigma_{L0} = \frac{\alpha c}{\Omega_{s0}} \sigma_{\epsilon 0}$ | $\frac{2\sqrt{\pi}}{\Gamma(1/4)} \left(\frac{3}{N^2 - 1} \right)^{1/4} \left(\frac{h\omega \sigma_{\epsilon 0}}{v_{s0}} \right)^{1/2} \frac{c}{\omega_{rf}}$ | $\sigma_{L0} \left(1 + \frac{Nk}{\cos\phi_s} \right)^{-1/2}$ |